PAPER 6

IMPROVEMENTS TO THE HYDRAULIC PERFORMANCE OF CULVERTS UNDER INLET CONTROL CONDITIONS THROUGH THE OPTIMISATION OF INLET CHARACTERISTICS

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ABSTRACT

With a growing focus on optimizing the hydraulic performance of both new and existing culverts, especially given South Africa's changing road network and expected shifts in rainfall patterns due to climate change, this study delved into the advantages of using angled wingwall and headwall combinations. It also explored the potential benefits of installing a ventilation device to enhance culvert performance. Experimental modelling conducted at the University of Pretoria Water Laboratory revealed that the angled wingwall and headwall configurations led to significant improvements in flow compared to traditional square inlets. Furthermore, the study discovered that a ventilation device could alter the flow dynamics within culverts, causing them to operate under inlet control conditions rather than outlet control conditions.

The research suggests adjustments to design coefficients for square inlet culverts operating under inlet control conditions, providing practical insights for enhancing culvert performance during the design phase. Additionally, the study proposes the use of prefabricated inlet elements as cost-effective solutions for upgrading existing culverts, offering a means to effectively improve performance without requiring lengthy road closures. It was found that the for varying degrees of modifications an increase in performance of between 16% and 18% could be achieved at optimum depth over height ratios when compared to the unmodified model results.

A practical implementation of these proposed modifications has been designed and will be monitored to evaluate the efficacy of the improvements. Overall, this study highlights the potential of innovative design modifications to boost culvert performance, offering sustainable and economical alternatives to conventional replacement practices. It contributes to advancing hydraulic engineering resilience in response to evolving infrastructural and environmental challenges.

1. INTRODUCTION

South Africa faces severe challenges in service delivery at the provincial and local government levels, largely due to poorly maintained infrastructure. This situation negatively impacts the economies of many communities, particularly in rural areas where the condition of provincial and municipal roads is visibly deteriorating. The South African National Roads Agency (SOC) Ltd. (SANRAL) are incorporating provincial roads into its national road network to support the country's medium to long-term developmental needs. However, many of these incorporated roads were not originally designed to meet the standards specified in the SANRAL Drainage Manual, leading to frequent overtopping of culverts during minor floods. This not only damages the structures but also poses a danger and inconvenience to road users. Additionally, climate change projections indicate an increase in the intensity of rainfall and extreme weather events in South Africa, which could further exacerbate the inadequacy of existing culvert designs.

As a result, SANRAL is mandated to replace various culverts in its current and future networks. Traditional culvert replacement involves significant traffic disruptions and capital investments. However, modifying existing culverts to improve their hydraulic performance may provide a cost-effective alternative. Such improvements could meet the required standards without necessitating costly road closures and reinstatements, thereby reducing overall project costs.

Despite the potential benefits, hydraulically optimized designs have not been widely adopted in culvert design. While there are numerous guidelines to assist engineers in selecting appropriate culvert sizes, the optimization of culvert inlets is often seen as risky, potentially leading to conservative design choices. Nonetheless, research into culvert inlet improvements suggests that these modifications can be confidently integrated into designs, offering benefits such as enhanced hydraulic performance, environmental sustainability, and cost-effectiveness.

Improving culvert inlets offers several other important benefits such as reduced risk of flooding (Sellevold and Norem (2023); Smith and Oak (2011)), enhanced sediment transport (Zayed (2023); Ho et al. (2013)), improved fish passage (Katopodis and Gottesfeld (2018); Arthur and Parola (2008)), extended culvert lifespan (Wagener and Leagjeld (2014); Norman et al. (2001)) and improved road safety (Thomson et al. (2006); Levine (2013)).

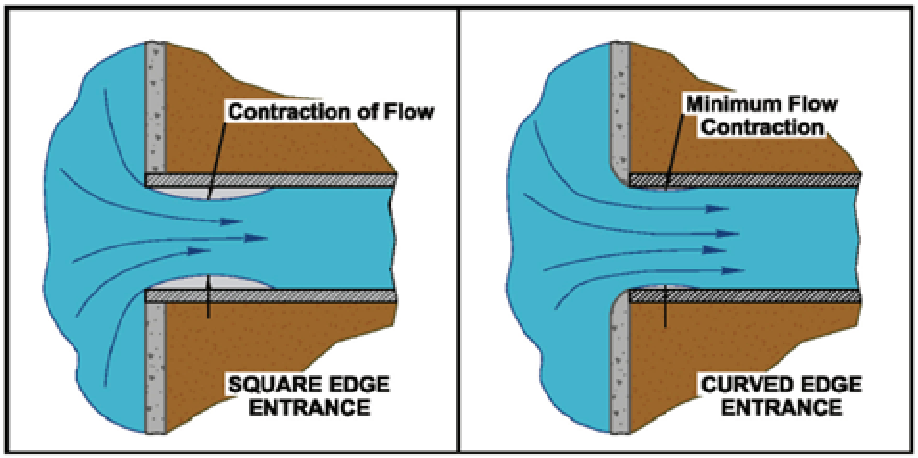

FIGURE 1: *Entrance contractions (James et al., 2012)*

TABLE 1: *Impact of culvert improvements (adapted from De Jager and Van Dijk, 2024)*

Researcher	Conclusions
Straub et al. (1953)	Noted that rounded inlets are advantageous over square inlets for culverts operating under inlet control.
West (1956)	This investigation evaluated the effects of inlet geometry upon the operation of culverts under inlet control to be able to predict the head loss at the entrance. The benefits of modifying the inlet characteristics to reduce head loss was described.
Hughes (1963)	Analysed the benefits of drop inlet-type culverts compared to standard box or pipe culverts and found these to have increased hydraulic capacity
French et al. (1966)	During this study it was found that culvert performance can be improved without altering the culvert slope and simply modifying the various wingwall angles.
Harrison et al. (1972)	The authors found in this study that bevelled edges increase culvert capacity by 5% to 20%, while side-tapered inlets provide a 25% to 40% increase in flow. Slope-tapered inlets can increase the capacity of conventional culverts with square edges by over 100%
Graziano et al. (2001)	Model studies were conducted and it was found that a cast-in-place, 30° flared-wingwall inlet is approximately 8% more efficient than a similar model with a 0° flared inlet under unsubmerged conditions. Under submerged conditions, the 30° flared wingwall resulted in a 10% lower H/D ratio than the 0° flared inlet.
Kerenyi et al. (2005)	During the conduction of model studies, it was found that a culvert with a top bevel radius of 203 mm was more efficient than one with a 102 mm radius or a square-edged bevel at the crown, which was the least efficient. The authors also noted a significant hydraulic advantage for multiple culvert barrels over single barrels for submerged flow, especially for headwater depths of 1.5 times the culvert height, when using precast models with the optimum bevel on the top plate.
Jones et al. (2006)	The following findings and conclusions were made from this experimental modelling study: - A radius top bevel edge was found to be the optimal shape among the six shapes tested, significantly improving culvert performance. This improvement was more pronounced for multiple barrels at higher headwater depths. - A 45° straight top bevel edge performed better than a square top edge with zero-degree wingwall flare edges. - Rounded bevels for wingwall top edges had no noticeable impact on performance. - The size of corner fillets, which are sometimes specified to minimize high-stress areas in the corners of rectangular culverts, had no discernible effect on culvert performance as long as the net culvert area was used in the discharge calculation. - The utilization of bevelled edges at the entrance of the culvert has been shown to be effective to increase the inlet performance as the bevelled edges reduce the contraction of flow by effectively enlarging the face of the culvert, as shown in Figure 1.
Ashour et al. (2014)	This study discovered that the angle of entrance headwall inclination enhances the discharge efficiency of both circular and box culverts compared to projected culverts of similar dimensions, with the greatest improvement observed at a 15° angle in the opposite direction of the stream. For circular culverts, this improvement (under inlet control) was found to be 6.7%.
Jaeger (2019)	Conducting computational fluid dynamics modelling and experimental flume tests, it was found that modifying culvert inlet corners can significantly enhance flow rates. The study found that large, rounded inlets or 45° chamfers performed best, while inlet angles of 30° and 60° caused more turbulence than 45°. Specifically, a rounded inlet corner with a radius of 0.15 times the culvert diameter could improve the flow rate by up to 20% while maintaining constant headwater levels.
Jaeger et al. (2019a)	The authors stated that the sudden reduction in cross-sectional flow area at the inlet, where an open channel enters the culvert, determines the flow through the culvert, even though the actual culvert barrel could convey higher flow rates.
Jaeger et al. (2019b)	During numerical and physical modelling, it was found that altered inlet corners can significantly improve flow rates in pipes, with large, rounded inlets or 45° chamfers performing best during simulations. The study also found that inlet design is one of the restricting factors in culvert flows.

In early research, the benefits of well-designed inlets were already recognized as summarised in Table 1.

It could be required that an existing culvert needs to be re-evaluated in terms of the flood for which it has been originally designed. A culvert's design flow rate could be adjusted upwards due to various factors, including changes in its catchment, the effects of climate change or due to the reclassification of the road it serves (requiring the culvert to convey floods with a higher return period).

If an existing culvert's hydraulic capacity has been calculated (by calculating the upstream energy head) and found to be insufficient for the new required design flow rate, the capacity of the culvert will have to be increased by:

- Replacing the culvert with one with a higher hydraulic capacity;
- Installing additional culverts in parallel;
- Changing the vertical alignment of the road to increase the allowable upstream energy head (H) (According to the SANRAL Drainage Manual (Kruger et al. 2013) allows the level of H to be in excess of 1.2 D only under specific conditions);
- Attenuating the flood to mitigate the need for increasing capacity of the culvert or
- Optimising the inlet of the culvert to improve the hydraulic efficiency for culverts operating under inlet control.

These options should be carefully considered and options which are impractical, or which will not be allowed should be discarded. It has been found that improving the hydraulic efficiency of culverts by retrofitting existing culverts offers a lower cost and time efficient alternative compared to the replacement or rebuild of infrastructure (Jaeger, 2019). Additionally, it may reduce the need to close roads for major construction works, preventing additional economic impacts.

2. HYDRAULIC ANALYSIS OF CULVERTS

Culvert hydraulics are well defined by the two conditions which govern the flow through the culvert barrel. Conveniently, these two conditions are named after the position where the dominant variables which influence the head required to push the water through the culvert can be found, namely inlet and outlet control conditions.

Inlet Control occurs at steep culverts and the flow in the culvert is only limited by the size, shape and configuration of the inlet. It is the sudden reduction of the cross-sectional flow area at the inlet where an open channel enters the culvert that determine the flow through the culvert, even though the actual culvert barrel could convey higher flow rates (Jaeger et al., 2019a). With *Inlet Control*, flow goes through critical depth near the inlet and downstream disturbances

TABLE 2: *Equations for calculating the flow through culverts under inlet control*

Reference	Type	Equation	Limitations
SANRAL (2013)	Round Culverts	$\frac{Q}{D^2\sqrt{gD}} = 0.48\left[\frac{S_0}{0.4}\right]^{0.05}\left[\frac{H_1}{D}\right]^{1.9}$ (Equation 1)	$H_1/D < 0.8$
		$\frac{Q}{D^2\sqrt{gD}} = 0.44\left[\frac{S_0}{0.4}\right]^{0.05}\left[\frac{H_1}{D}\right]^{1.5}$ (Equation 2)	$0.8 < H_1/D < 1.2$
		$Q = C_D A\sqrt{2g\left(H_1 - \frac{D}{2}\right)}$ (Equation 3)	$H_1/D \geq 1.2$
	Rectangular Culverts	$Q = \frac{2}{3}C_B BH_1\sqrt{\frac{2}{3}gH_1}$ (Equation 4)	$H_1/D \leq 1.2$
		$Q = C_h BD\sqrt{2g(H_1 - C_h D)}$ (Equation 5)	$H_1/D > 1.2$
Schall et al. (2012)	Unsubmerged Equation Form 1	$\frac{H_1}{D} = \frac{H_c}{D} + K\left[\frac{1.811\,Q}{AD^{0.5}}\right]^M + K_s S_o$ (Equation 6)	$\frac{Q}{AD^{0.5}} < 1.93$
	Unsubmerged Equation Form 2	$\frac{H_1}{D} = K\left[\frac{1.811\,Q}{AD^{0.5}}\right]^M$ (Equation 7)	$\frac{Q}{AD^{0.5}} < 1.93$
	Submerged	$\frac{H_1}{D} = c\left[\frac{1.811\,Q}{AD^{0.5}}\right]^2 + Y + K_s S_o$ (Equation 8)	$\frac{Q}{AD^{0.5}} > 2.21$
Marek & Marek (2009)	Fifth-degree polynomial equation	$H_1 = [a + bF + cF^2 + dF^3 + eF^4 + fF^5]D - 0.5DS_0$ (Equation 9) Where: $F = \frac{1.811\,Q}{BD^{\frac{3}{2}}}$ (Equation 10)	$0.5 \leq \frac{H_1}{D} \leq 3$
	Orifice Equation	$H_1 = \left[\frac{Q}{k}\right]^2 + \frac{D}{2}$ (Equation 11) Where: $k = \frac{0.6325Q_3}{D^{\frac{1}{2}}}$ (Equation 12)	$\frac{H_1}{D} > 3$
Charbeneau (2005)	Unsubmerged conditions	$\frac{H_1}{D} = \frac{3}{2}\left(\frac{1}{C_e}\right)^{\frac{2}{3}}\left(\frac{Q}{BD\sqrt{gD}}\right)^{\frac{2}{3}}$ (Equation 13)	$\frac{H_1}{D} < \frac{3}{2}C_f$
	Submerged Conditions	$\frac{H_1}{D} = \frac{1}{2C_g{}^2}\left(\frac{Q}{BD\sqrt{gD}}\right)^2 + C_f$ (Equation 14)	$\frac{H_1}{D} > \frac{3}{2}C_f$

are not propagated upstream where flow is supercritical in the culvert barrel (Jones et al., 2006).

Outlet Control occurs for mild slope culverts where free surface flow is subcritical and for any slope where the barrel is completely submerged. In these cases, the tailwater is the control (Jones et al., 2006). Under these conditions, the barrel of the culvert contributes to the head loss, and therefore calculations of outlet control incorporate parameters from inlet control as well as the length and material of the culvert and the tailwater height (Jaeger et al., 2019a).

It is useful to keep in mind that inlet control occurs when the flow capacity of the culvert entrance is less than the flow capacity of the culvert barrel. The culvert entrance therefore controls the headwater elevation for a given flow. Similarly, outlet control occurs when the culvert flow capacity is limited by downstream condition or by the flow capacity of the culvert barrel (Brunner et al., 2018).

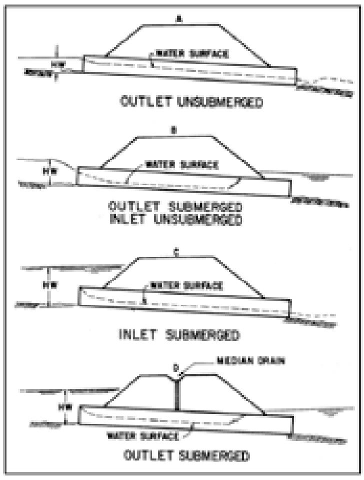

FIGURE 2: *Types of inlet control (Norman et al., 2001)*

Inlet control occurs most often and is preferred since it yields the smallest culvert cross-section for a given upstream head and the higher flow velocities through the culvert barrel prevents the deposition of sediment inside the culvert (SANRAL, 2013). Several different examples of inlet control are depicted in Figure 2 (Norman et al., 2001).

Sketch A in Figure 2 depicts a condition where neither the inlet nor the outlet end of the culvert is submerged, flow passes through critical depth just downstream of the culvert entrance with supercritical flow occurring in the barrel. In sketch B and D the submergence of the outlet end of the culvert does not result in outlet control, as there is still a hydraulic jump that forms in the barrel. Sketch C is the most typical inlet control design situation and also represents the focus of this study.

Table 2 summarises selected equations for calculating the flow through culverts under inlet control conditions.

Where:

H_1 = headwater depth above inlet control section invert (m)
Q = discharge (m^3/s)
D = inside diameter (m) or height (inside) (m)
A = full cross-sectional area of culvert barrel (m^2)
S_0 = slope of culvert bed (m/m)
C_D ≈ 0.6
B = width (inside) (m)
C_B = 1 for rounded inlets ($r > 0.1B$) and
C_B = 0.9 for square inlets
C_e, C_f, C_g = representative parameter values for culvert performance
C_h = 0.8 for rounded inlets and
C_h = 0.6 for square inlets.
H_c = specific head at critical depth (m) $\left(d_c = \frac{V_c^2}{2g}\right)$

K, M, c, Y = constants
g = standard gravity (9.81 m/s^2)
K_s = slope correction, -0.5 (mitred inlets +0.7)
a, b, c, d, e, f = regression coefficients
F = function of average outflow discharge routed through a culvert

As an example, to show the variations obtained when using the different formulae performance curved were setup using the following parameters for a box culvert: D = 1.8 m; S_0 = 0 m/m; C_D = 0.6; B = 1.8 m; C_b = 0.9; C_h = 0.6; M = 2; K = 0.0083; K_{uS} = -0.01; a = 0.144138; b = 0.461363; c = -0.092150; d = 0.020003; e = -0.001360; f = 0.000036; C_e = 1; C_f = 0.667; C_g = 0.667; and g = 9.81 m/s^2 and depicted in Figure 3.

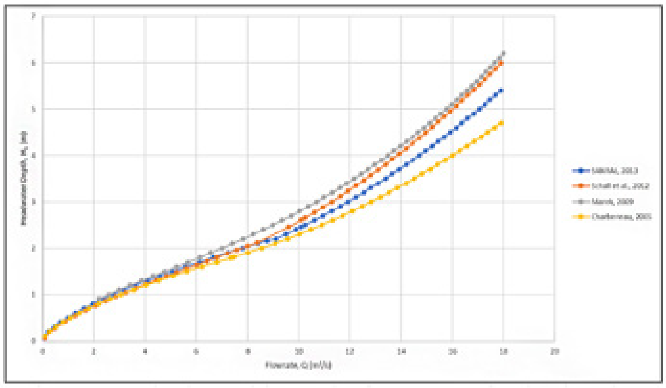

FIGURE 3: *Comparison between inlet control performance curves for selected equations*

The Federal Highway Administration (Schall et al., 2012) states that: "The most widely recognized manual on culvert hydraulics is the FHWA Hydraulic Design Series No. 5 (HDS-5), Hydraulic Design of Highway Culverts, published in 1985, but based on research conducted in the 1960s and 1970s." This statement suggests that there is scope for further research into the optimisation of culverts in general. In order to provide guidelines for practitioners on the benefits of improving the inlet characteristics of culverts an experimental model was constructed to quantify the benefits that could be obtained from such modifications.

3. PHYSICAL MODELLING OF CULVERTS

An experimental culvert model was constructed in the Water Laboratory of the University of Pretoria (UP Waterlab). This model featured a single-barrel square culvert with three different headwall/wingwall (inlet) combinations. The model channel and culvert barrel were made of clear Plexiglass with a thickness of 10mm, and two of the three culvert inlets were 3-D printed. The model's frame was hinged on one side and equipped with two hydraulic jacks on the other to allow for adjustment of the culvert slope. The setup was supplied with two BADU Porpoise 22 1.1kW pumps installed in parallel and a mechanical flow meter on the upstream of the model. To ensure uniform flow towards the culvert inlet and reduce wave action, PVC conduits were cut into 200mm lengths, glued together, and installed on the upstream side of the model channel. A schematic flow diagram of the constructed model is shown in Figure 4.

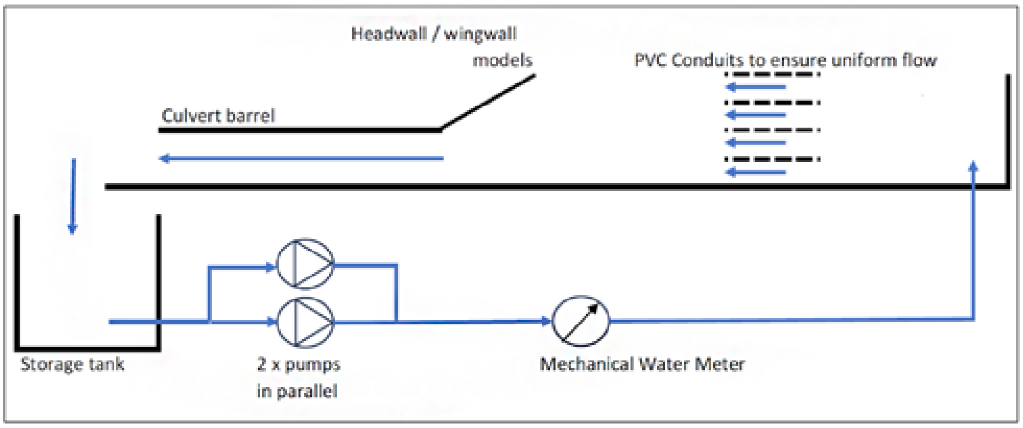

FIGURE 4: *Schematic flow diagram of model (De Jager and Van Dijk, 2024)*

FIGURE 5: *Physical model (flume with culvert model)*

Three angles for the headwall and wingwalls were selected, namely 90°, 45° and 30°. Both the wingwalls and headwalls were positioned at these angles, measured parallel with the inside of the culvert walls, opposite to the direction of flow. Since these configurations for the 45° and 30° models created complex shapes, these models were 3D printed (Figure 6) so that their inlets could fit inside of a collar which was provided on the plexiglass culvert section.

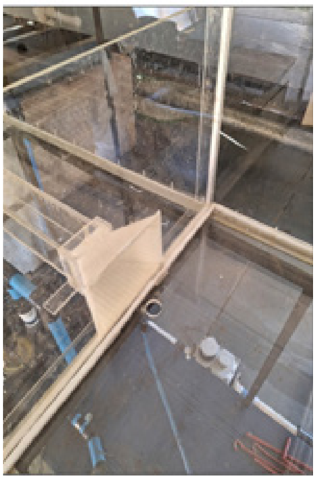

FIGURE 6: *Culvert model showing 3D printed inlet*

The first step was to evaluate the model results and compare with the available formulae. The measured headwater depths were plotted against the average flow rates and a fifth-degree polynomial trendline was generated using the plotted data. The measured data and the trendline for the 90° model as well as inlet control performance curves for selected equations is depicted in Figure 7.

For unsubmerged flow, the results fit the performance curve for Shall et al., 2012 the best. For submerged flow (from about $H_1/D > 1.2$) the trendline tracks between Kruger et al. (2013) and Charbeneau (2005). The physical model thus showed good correlation with the theoretically determined values.

The experimental results for each headwall/wingwall model experiment showed encouraging

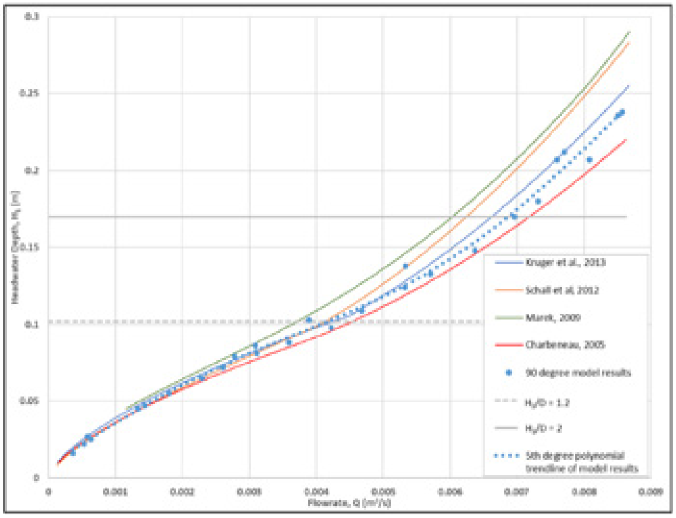

FIGURE 7: *Comparison of model results (90° model; Kruger et al. (2013); Schall et al. (2012); Marek & Marek (2009) and Charbeneau (2005)),* adapted from De Jager and Van Dijk (2024)

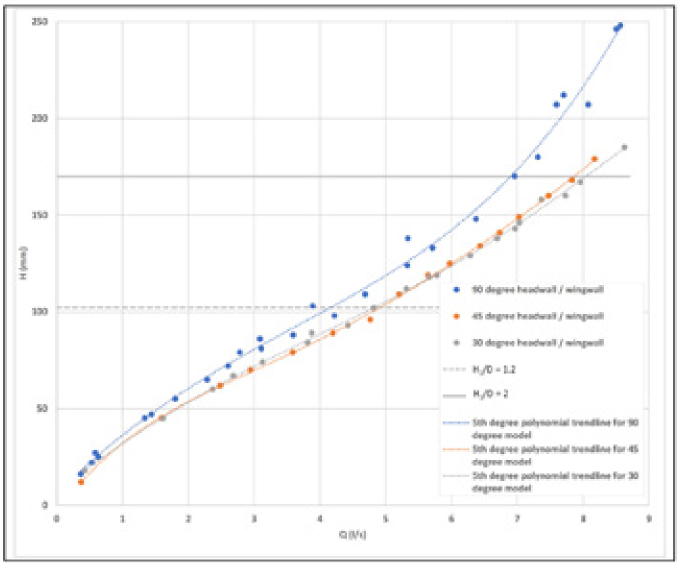

FIGURE 8: *Comparison between results for the three headwall/wingwall models (adapted from De Jager and Van Dijk (2024))*

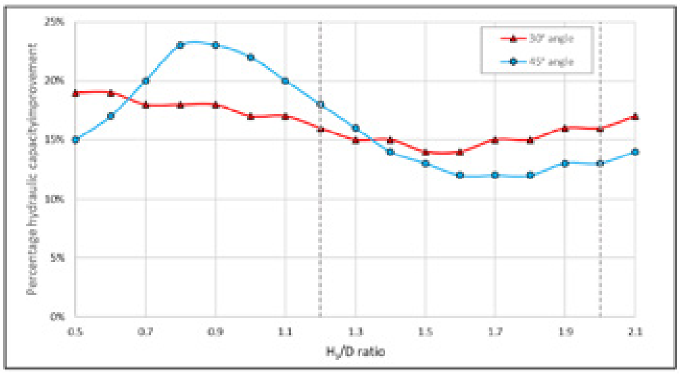

FIGURE 9: *Performance improvement of modified culverts compared with unmodified*

hydraulic improvements that could be obtained from modifying the inlet to have an angled headwall and wingwall. The results of the conducted tests are depicted in Figure 8.

The improved hydraulic capacity for the various H_1/D ratios and flow rates are presented in Figure 9. Figure 9 shows that, even though there is a significant increase in the performance of culverts when angled headwalls and wingwalls are installed (when compared to square inlets), there isn't a significant increase in performance between the 30° and 45° models at the typical design H_1/D ratio of 1.2 (16% compared with 18%). This correlates to the word done by Marek in 2009, where the same parameters were given for the 5th order polynomial equations for flared wingwalls between 30° and 70° (Marek and Marek, 2009).

4. PRACTICAL IMPLEMENTATION OF INLET CHARACTERISTIC IMPROVEMENTS

Improving the inlet characteristics of a culvert system is crucial for enhancing its hydraulic capacity and ensuring efficient water flow management. Practical implementation involves several strategies: reshaping the inlet to minimize flow contraction and turbulence, installing headwalls and/or wingwalls to guide the flow smoothly into the culvert, and incorporating bevels or flares to expand the inlet area and reduce entrance losses. These modifications can significantly reduce energy losses and increase the flow capacity. Additionally, using more hydraulically efficient materials and regular maintenance to remove debris can prevent blockages and maintain optimal flow conditions. Implementing these inlet improvements requires a detailed hydraulic analysis to determine the most effective design modifications for specific site conditions, ensuring that the culvert system operates at its maximum efficiency and capacity. An example of the implementation of a shaped inlet is shown in Figure 10 which is a culvert at the confluence of three stream draining three catchments (Hoogekraal, Seekat Road and Oudeweg) in Glentana, Southern Cape. Floods in 2006 washed away most of the stormwater infrastructure and the construction of a new culvert system with a modified inlet provided sufficient hydraulic capacity.

FIGURE 10: *Improved inlet configuration (Glentana, Southern Cape)*

As part of a case study a road culvert's inlet characteristics will be modified and evaluated over a period to evaluate its performance. The culvert located is in the Umbilo River and passes under the N3 National Road near Durban in Kwa-Zulu Natal. This culvert is located between 10-15m under the road surface. Several historical rain events have caused flooding upstream of the culvert. Increasing the culvert's hydraulic performance sufficiently by

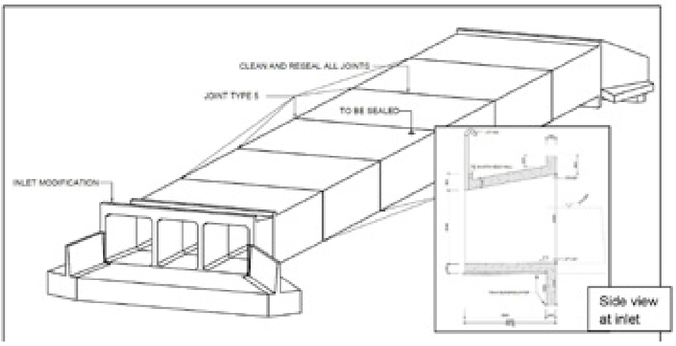

FIGURE 11: *Modified inlet design (Umbilo River culvert system)*

optimising the inlet parameters (preliminary improvements that have been suggested include the optimisation of wing walls, tapered head wall and potentially the provision of an air vent) may negate the need for major roadworks (and associated significant disruption of traffic) to replace the culvert.

5. CONCLUSIONS & RECOMMENDATIONS

The design of culverts is a fundamental topic in engineering hydraulics courses globally, yet the standard design approaches often neglect the potential benefits of optimizing inlet characteristics for culverts under inlet control conditions. This oversight, coupled with some designers' overly conservative approaches, has led to many culverts being overdesigned and unnecessarily expensive. However, there is a renewed interest in this area, which is promising. SANRAL's incorporation of provincial roads into its national network requires these roads to meet more stringent design criteria, often necessitating culverts that can handle higher design floods than originally intended. Additionally, climate change projections indicate an increase in extreme rainfall events, potentially rendering the original culvert designs insufficient.

Addressing these issues in new culverts is straightforward, but increasing the capacity of existing culverts poses a significant challenge. Traditional methods involve road closures and replacing or adding new culverts, which can be costly and inconvenient. An effective alternative could be for some systems to enhance the capacity of the existing culverts in situ. This research focused on improving hydraulic performance by adding angled wingwalls and headwalls to culvert inlets. Using an experimental model at the University of Pretoria Water Laboratory, different inlet combinations were tested, revealing significant performance improvements.

The experimental model included a single-barrel square culvert and three headwall/wingwall combinations (90°, 45° and 30°). Results showed that the 45° and 30° models improved flow rates significantly compared to the 90° model, with performance increases of up to 18% and 16%, respectively (at H_1/D ratios of 1.2). This study demonstrated that precast headwall/wingwall elements could be easily attached to existing culverts, offering a cost-effective solution that enhances hydraulic performance without extensive road disruptions. This research project funded by SANRAL aims to provide design guidelines, incorporated into the SANRAL Drainage Manual, on various improvements that can be considered to improve the hydraulic capacity of culvert systems.

This study has identified the need for further research to be conducted:

- A detailed study into the potential benefits of installing a ventilation device in culverts to ensure a free surface in the culvert;
- The increase in efficiency of an angled headwall inlet improvement over angled wingwalls only should be investigated. The experimental work included both improvements but to identify each individual improvement's contribution would be valuable.
- It is recommended that a Computational Fluid Dynamics model be developed, incorporating this study, to aid in future culvert flow calculations and to assist with the analyses of multiple variations. This will aid in the compilation of design graphs that could more easily be used by design engineers to incorporate this into their designs.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Arthur JR and Parola C 2008. Pool Simulation Culvert Design for Fish Passage. In Proceedings of the World Environmental and Water Resources Congress, Honolulu, HI, USA, 12 May 2008.

Ashour MA, Aly TE and Abdallah AA 2014. Inclined headwall is an efficient tool for maximizing the discharge efficiency through culverts. In Proceedings of the 3rd International Conference - Water Resources and Wetlands, Tulcea, Romania, 9–13 September 2014.

Brunner GW Sanchez, A, Molls T and Parr DA 2018. HEC-RAS Verification and Validation Tests. US Army Corps of Engineers Institute for Water Resources Hydrologic Engineering Centre. Research Document, report number RD-52. Davis, California.

Charbeneau RJ 2005. Hydraulics of Low-Headwater Box Culverts. Project Summary Report Number 0-2109-S. Centre for Transportation Research, The University of Texas at Austin: Austin, TX, USA.

De Jager L and Van Dijk M 2024. Improvements to the Hydraulic Performance of Culverts under Inlet Control Conditions by Optimisation of Inlet Characteristics. Water, 16,1569.

French JL 1966. Sixth Progress Report on Hydraulics of Culverts. Tapered Box Culvert Inlets. Report number 9355. Bureau of Public Roads, US Department of Commerce: Washington, DC, USA.

Jaeger R. 2019. Hydraulic Improvements in Culverts for Climate Change Adaptation. Ph.D. Thesis, University of the Sunshine Coast, Queensland, Australia.

Jaeger R, Tondera T, Pather S, Porter M, Jacobs C and Tindale N 2019a. Flow Control in Culverts: A Performance Comparison between Inlet and Outlet Control. MDPE Water - Open Access Journal., Vol 11, 1408.

Jaeger R, Jacobs C, Tondera T and Tindale N 2019b. Improving flows in misaligned culverts. MDPE Water -Open Access Journal, Vol 11, 1932.

James D, Schal P, Thompson S and Zerges R 2012. Hydraulic design of highway culverts. Report No. FHWA-HIF-12-026. Office of Bridge Technology.

Jones JS, Kerenyi K and Stein S 2006. Effects of Inlet Geometry on Hydraulic Performance of Box Culverts. Report number FHWA-HRT-06-138. The US Department of Transportation Federal Highway Administration: Springfield, VA, USA.

Graziano F, Stein S, Umbrell E and Martin B 2001. South Dakota Culvert Inlet Design Coefficients. Report number FHWA-RD-01-076. The US Department of Transportation Federal Highway Administration: Springfield, VA, USA.

Harrison LJ, Morris JL, Norman, JM and Johnson FL 1972. Hydraulic Design of Improved Inlets for Culverts. Report FHWA/EO-72-13. Hydraulic Engineering Circular No. 13 (HEC13). Federal Highway Administration: Washington, DC, USA.

Ho H, Muste M and Ettema R 2013. Sediment self-cleaning multi-box culverts. Journal of Hydraulic Research, Vol 51, pp. 92–101.
Hughes RD 1963. A Study of The Hydraulics of Drop-Inlet-Type Culvert Models. Commonwealth of Kentucky, Department of Highways, Highway Materials Research Laboratory: Lexington, KY, USA.
Katopodis C and Gottesfeld AS 2018. Fish Passage Engineering: Design and Application Methods. John Wiley & Sons: Hoboken, NJ, USA.
Levine J 2013. An Economic Analysis of Improved Road-Stream Crossing. The Nature Conservancy, Adirondack Chapter: Keene Valley, NY, USA.
Kerenyi K, Jones JS, Goeden K, Phillips R and Oien P 2005. A Better Design for Box Culverts? Report number FHWA-HRT-05-007. The US Department of Transportation Federal Highway Administration: Springfield, VA, USA. Volume 69, pp. 46–52.
Kruger EJ, Gomes N, Rooseboom A, Van Vuuren SJ, Van Dijk M, Jansen van Vuuren AM, Smithers JC, Pienaar WJ, Pienaar, PA, James GM, Maastrecht J and Stipp DW 2013. The South African National Roads Agency SOC Limited Drainage Manual, 6th ed. SANRAL: Pretoria, South Africa.
Marek A and Marek PE 2009. Hydraulic Design Manual. Manual Notice 2009-1. The Texas Department of Transportation: Austin, TX, USA.
Norman JM, Houghtalen RJ and Johnston WJ 2001. Hydraulic Design of Highway Culverts. National Highway Institute: Arlington, VA, USA.
Schall JD, Thompson PL, Zerges SM, Killgore RT and Morris JL 2012. Hydraulic Design of Highway Culverts, 3rd ed. Report FHWA-HIF-12-026 HDS 5. Ayres Associates: Fort Collins, CO, USA.
Sellevold J, Norem H, Bruland O, Ruther N and Pummer, E 2024. Effects of Bottom-Up Blockage on Entrance Loss Coefficients and Head-Discharge Relationships for Pipe Culvert Inlets: Comparisons of Theoretical Methods and Experimental Results. Journal of Irrigation and Drainage engineering, 150, 04023038.
Smith CD and Oak AG 2011. Culvert Inlet Efficiency. Canadian Journal of Civil Eng, 22(3), pp. 611–616.
Straub LG, Anderson, AG and Bowers CE 1953. Importance of Inlet Design on Culvert Capacity. University of Minnesota: Minneapolis, MN, USA.
Thomson R, Fagerlind H, Martinez AV, Amenguel A, Naing C, Hill J, Hoschopf H, Dupré G, Bisson O, Kelkka M, et al. 2006. Roadside Infrastructure for Safer European Roads: D06 European Best Practice for Roadside Design: Guidelines for Roadside Infrastructure on New and Existing Roads. Loughborough University: Loughborough, UK.
Wagener BD and Leagjeld EE 2014. Culvert Repair Best Practices, Specifications and Special Provisions: Best Practices Guidelines. Minnesota Department of Transport Research Services & Library: St. Paul, MN, USA.
West EM 1956. Hydraulic Model Studies of Culvert Operations. Commonwealth of Kentucky, Department of Highways, Highway Materials Research Laboratory: Lexington, KY, USA.
Zayed M 2023. Blockage slope and ratio effects on box culvert hydraulics. Journal Discover Water, Vol 3 (23).